Piecewise Linear Functions: Cellular Phone Plans

Write a piecewise function that models the cellular phone billing plan. Then graph the function.

82. $60 per month buys 450 minutes. Additional time costs $0.35 per minute. (From section 2.4.)

Instructions:

1. DUE: Friday, March 6th, 11:59 pm. Late submissions will be accepted for documented, extenuating circumstances. If there are no extenuating circumstances, late submissions can still be submitted with the Exam 1 e-portfolio revisions for possible extra credit, but will not count toward the 10% e-portfolio grade.

2. Please download the file MTH 163 Piecewise Linear Functions.pdf. (In this same folder.)

3. Please rename the file using the following format: LASTNAME Piecewise Linear Functions

4. Then please input your answers into the renamed file, using the "Description of Problem" and "Solution" fields that are provided. Please use 3 paragraphs to answer this question. In the three paragraphs, please address the following:

Description of Problem field:

Paragraph 1 Please state the problem including the following:

1. Description of the situation.

2. Why is a piecewise linear function appropriate for modeling this situation? When answering this part, please keep in mind that there are 2 parts to the piecewise function. Please state what the 2 parts of the cell phone plan are that you will model later, in Paragraph 2, with a function.

3. Do you expect your phone cost to increase or decrease after 450 minutes? Why?
Solution field:

Paragraph 2 Please state your solution to the problem, including the following:

1. The function that you came up with to model the cellular phone plan. Please explain what each part represents, in terms of the cellular phone plan, including the following:
   
   (a) The part of the function that corresponds to $60 for the first 450 minutes.
   (b) The part of the function that corresponds to $0.35 for each additional minute after 450 minutes.

Paragraph 3 Please upload/provide the graph of your piecewise function. Also, please address the following:

1. How do you know that your graph represents a function?

2. Please explain where each part of your piecewise function shows up on the graph that you provided. (There are 2 parts to the piecewise function.)

3. What is the rate at which your plan cost increases or decreases after $t=450$ minutes? Is this consistent with what you expected in part 2(b) above?

4. There are several options for providing a graph:
   
   (a) The TCC library should have a scanner, as do most of the public libraries. You may also submit a readable screenshot of your graph.
   (b) Alternatively, Winplot is a free grapher for Windows users that can generate a jpeg/tiff/pdf file of your graph that you can upload to Blackboard.
   (c) You may also use a graphing utility of your choice to generate and upload an image file in jpeg, tiff, or pdf format.
Modeling Cell Phone Plans with Piecewise Functions.

Description of Problem:

In this problem, we are using a piecewise function to model a cell phone plan that cost $60 per month for 450 minutes, and an additional $0.35 per minute if you go over those initial 450 minutes.

A piecewise function is a function defined by two or more equations over a given domain. In other words, the function acts differently on each piece of the number line. This type of function is good for modeling a cellphone plan because it can give you a physical graph that shows the increasing rate of the plan after you have exceeded a given amount of minutes. The two parts of the graph are the constant cost of the initial minutes and the cost after you have exceeded those minutes. In this situation, after the first 450 minutes, which is represented by a horizontal line, the cost will increase by $0.35 per minute, which will be represented by a positive sloping line. Since the plan charges you per minute after a given amount of minutes, I expect my phone bill to increase constantly as I use each minute after 450.
Solution:

The total monthly cost of the cell phone plan, \( C \), is written as a function of the number of calling minutes, \( t \):

\[
C(t) = \begin{cases} 
60 & \text{if } 0 \leq t \leq 450 \\
60 + 0.35(t-450) & \text{if } t > 450 
\end{cases}
\]

The cost is $60 for up to and including 450 minutes.

The cost is $60 plus $0.35 per minute for anything above 450 minutes.

The graph above is a function because it passes the vertical line test, where any vertical line only intersects the graph at one point. In other words, no value of \( t \) is paired to more than one value of \( C(t) \).

The first part of this piecewise function, where the cost for the cell phone plan is $60 for up to and including 450 minutes is represented by the horizontal line portion of the graph. The second part of this piecewise function, where the total cost of the plan is $60 plus $0.35 per minute for anything above 450 minutes is represented by the positive slope line.

The slope of \( C(t) = 60 + 0.35(t-450) \), or the rate at which the plan increases after \( t = 450 \) minutes, is 0.35. This constantly increasing slope agrees with my predictions from part 1.
Student Questions / Instructor Comments:

GRADE: 100%
Great work! Thank you for paying attention to detail, and for putting in the time to give such a polished presentation. This particular ePortfolio submission really stands out!